$F_3 : \quad \forall x \forall y \forall z (Rxx \wedge ((Rxy \wedge Ryz) \Rightarrow Rxz) \wedge (Rxy \Rightarrow Ryx))$
$F_4 : \quad \forall x \forall y \forall z (Rxx \Rightarrow R\, x * z\ y * z)$
$F_5 : \quad \forall x \forall y (Rxy \Rightarrow \neg Ryx).$

**8.** The language $L$ consists of a single binary predicate symbol, $R$.

Consider the $L$-structure $\mathcal{M}$ whose base set is $M = \{n \in \mathbb{N} : n \geq 2\}$ and in which $R$ is interpreted by the relation 'divides', i.e. $\overline{R}$ is defined for all integers $m$ and $n \geq 2$ by the condition: $(m, n) \in \overline{R}$ if and only if $m$ divides $n$.

(a) For each of the following formulas of $L$ (with one free variable $x$), describe the set of elements of $M$ that satisfy it.

$$F_1 : \quad \forall y (Ryx \Rightarrow x \simeq y)$$
$$F_2 : \quad \forall y \forall z ((Ryx \wedge Rzx) \Rightarrow (Ryz \vee Rzy))$$
$$F_3 : \quad \forall y \forall z (Ryx \Rightarrow (Rzy \Rightarrow Rxz))$$
$$F_4 : \quad \forall t \exists y \exists z (Rtx \Rightarrow (Ryt \wedge Rzy \wedge \neg Rtz)).$$

(b) Write a formula $G[x, y, z, t]$ of $L$ such that for all $a$, $b$, $c$ and $d$ of $M$, the structure $\mathcal{M}$ satisfies $G[a, b, c, d]$ if and only if $d$ is the greatest common divisor of $a$, $b$ and $c$.

(c) Let $H$ be the following closed formula of $L$:

$$\forall x \forall y \forall z ((\exists t (Rtx \wedge Rty) \wedge \exists t (Rty \wedge Rtz)) \Rightarrow \exists t \forall u (Rut \Rightarrow (Rux \wedge Ruz))).$$

(1) Find a prenex form of $H$.
(2) Is the formula $H$ satisfied in $\mathcal{M}$?
(3) Give an example of a structure $\mathcal{M}' = \langle M', \overline{R} \rangle$ such that when $\mathcal{M}$ is replaced by $\mathcal{M}'$ in the previous question, the answer is different.